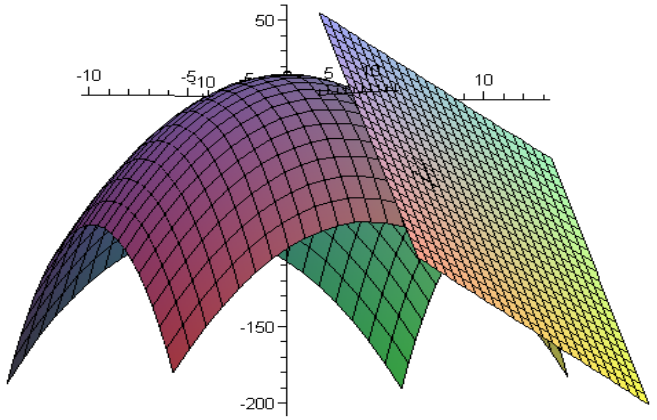
14.4 Tangent Planes



Entry Task: *HW Q1–4*

Consider $z = f(x,y) = 15 - x^2 - y^2$
at the point $(x_0, y_0) = (7,4)$.
Find/compute the following:

(a) The corresponding "height", z_0.

(b) The slope in the x-direction.

(c) A tangent vector in the x-direction.

(d) The tangent line in the x-direction.

The plane that contains all tangent lines to a given surface at a point is given by

Visual: https://www.desmos.com/3d/yprgwmf5ug

$$-f_x(x_0, y_0)(x - x_0) - f_y(x_0, y_0)(y - y_0) + (z - z_0) = 0$$

which we typically write as:

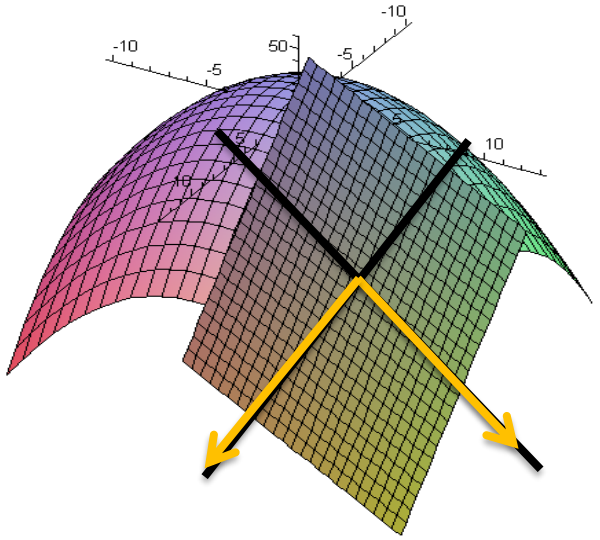
$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Example:

Find the tangent plane for

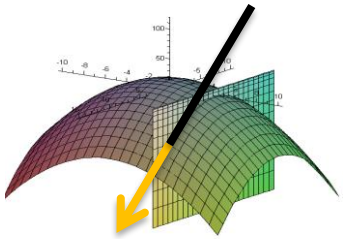
$z = f(x,y) = 15 - x^2 - y^2$

at the point (x,y) = (7,4)

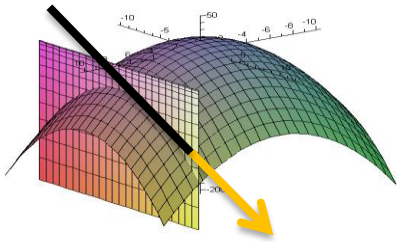


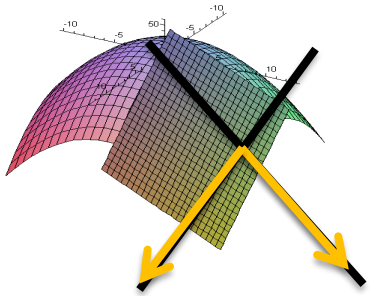
Example Derivation

$$f_x(x,y) = -2x \quad , \quad f_x(7,4) = -14$$



$$f_y(x,y) = -2y \quad , \quad f_y(7,4) = -8$$





Thus, we can get two vectors that are parallel to the plane:

$$\langle 1, 0, f_x(x_0, y_0)\rangle = \langle 1, 0, -14\rangle$$
$$\langle 0, 1, f_y(x_0, y_0)\rangle = \langle 0, 1, -8\rangle$$

A normal vector is given by

$$\langle 1, 0, -14\rangle \times \langle 0, 1, -8\rangle = \langle 14, 8, 1\rangle$$

Tangent Plane:

$$14(x-7) + 8(y-4) + (z+50) = 0$$

$\langle 1,0, f_x(x_0, y_0)\rangle$ = 'tangent in *x*-dir.' $\langle 0,1, f_y(x_0, y_0)\rangle$ = 'tangent in *y*-dir.'

Upward pointing normal to the surface

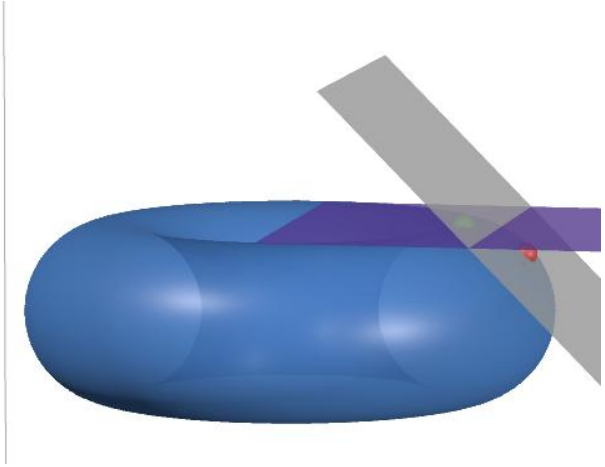
$$\langle 1,0, f_x(x_0, y_0)\rangle \times \langle 0,1, f_y(x_0, y_0)\rangle = \langle -f_x(x_0, y_0), -f_y(x_0, y_0), 1\rangle$$

Example: Tangent Planes on a Donut

Consider the torus (donut-shaped surface)

$$\left(\sqrt{x^2+y^2}-2\right)^2+z^2=1$$

The points $P(2,0,1)$ and $Q\left(2+\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}\right)$ are on this torus (why?).



Find the tangent planes at these points.

https://www.desmos.com/3d/ekg8pmmhw0



An Application of the Tangent Plane

Linear Approximation *HW Q5–6*

"Near" the point (x_0,y_0) the tangent plane and surface z-values are close.

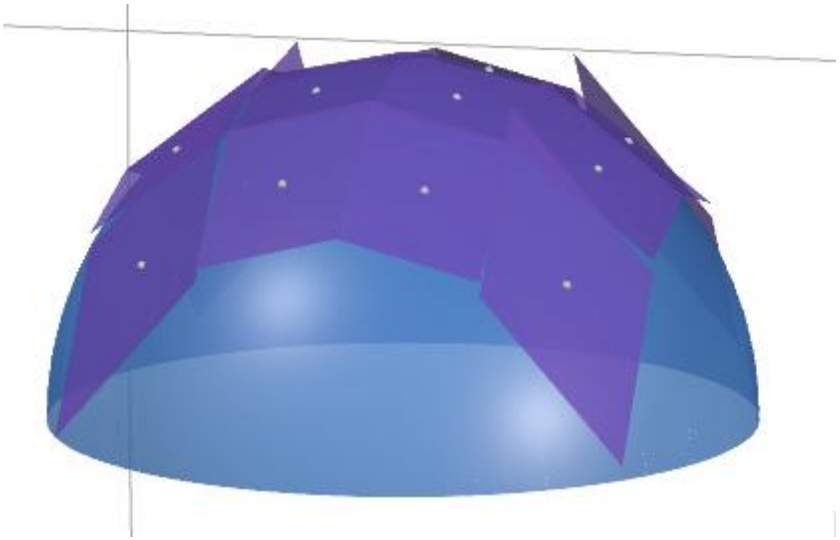
$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0),$$

which is the same as

$$L(x, y) = z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

Idea:

$$f(x, y) \approx L(x, y) \text{ for } (x, y) \approx (x_0, y_0)$$





https://www.desmos.com/3d/hj4zzyphnj

Example: Building Solid Cylinders

A company builds solid circular cylinders with radius 2 meters and height 10 meters.

The formula for the total surface area of a cylinder is $S(r,h) = 2\pi r^2 + 2\pi rh$

(a) Find the tangent plane for $S(r,h)$ at $(r,h) = (2,10)$.

Numerical References

$S(2,10) = 2\pi(2)^2 + 2\pi(2)(10) = 48\pi$

$S(2.1,9.8) = 2\pi(2.1)^2 + 2\pi(2.1)(9.8) = 49.98\pi$

$L(2,10) = 48\pi + 28\pi(0.1) + 4\pi(-0.2) = 50\pi$

(b) A collection of cylinders come off the assembly line with slight errors in manufacturing leading to a radius 2.1 meters and height 9.8 meters.